Dependence of the Angular Distribution of the (d,p) Reaction on the Total Angular-Momentum Transfer*

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Experimental evidence is presented that the angular distribution of the (d,p) reaction depends not only on the orbital angular momentum l but also on the total angular-momentum transfer $J=l\pm\frac{1}{2}$. For $l=1$, marked differences appear between $J=\frac{1}{2}$ and $\frac{3}{2}$ at angles greater than 90° in medium-weight nuclei. Lesser differences also appear for $l=2$ and $l=3$ transitions at back angles. A marked dependence on J is also observed for $l=2$ transitions at forward angles. The experimental evidence for such effects is presented.

I. INTRODUCTION

THE dependence of the angular distribution of (d,p) reactions on the orbital angular-momentum transfer was first pointed out by Butler.[1] This simple feature of the (d,p) reaction has made it an extremely useful tool in nuclear spectroscopy. A great deal of the information which has been accumulated regarding the spins of nuclear energy levels has been derived from experimental measurements of (d,p) angular distributions.

The orbital angular-momentum transfer deduced from a (d,p) angular distribution does not, in general, provide enough information to allow one to deduce the total spin of the final state. At best, when the target nucleus has zero spin, the final spin is limited to $J=l\pm\frac{1}{2}$. The remaining ambiguity can only be resolved by further measurements. The most reliable method of determining the spin of the final state is to study the electromagnetic radiation emitted in its decay. The measurement of the angular correlation of these γ rays can frequently remove the ambiguity in J, although sometimes additional information on lifetimes and internal-conversion coefficients is needed. Other, but less reliable, methods for removing the $\pm\frac{1}{2}$ ambiguity in J are the study of polarization in the (d,p) reaction or the comparison of intensities in the (d,p) and (d,t) reactions leading to the same final states. The case of target nuclei of nonzero spin is, of course, more complicated.

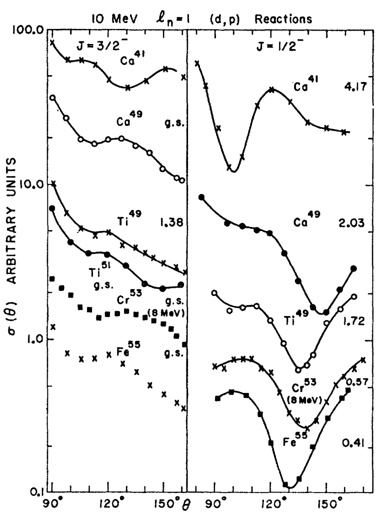

FIG. 1. Angular distributions in the backward direction for the (d,p) reaction. These are for $l=1$ transitions to final states of known spin, except for Ca^{49} for which the spins are inferred from the simple single-particle-like level structure. The measurements on C rare those of Ref. 8.

II. J DEPENDENCE IN $l=1$ TRANSITIONS

In a recent note[2] it was pointed out that in medium-weight nuclei the (d,p) angular distributions for $l=1$, $J=\frac{1}{2}$ states are different from those for $l=1$, $J=\frac{3}{2}$ states. The experimental data for ten states with $J=\frac{1}{2}$ and eleven with $J=\frac{3}{2}$ are shown in Figs. 1 and 2. All $J=\frac{1}{2}$ states show a sharp minimum at some angle between 90° and 140°, but none of the $J=\frac{3}{2}$ states shows such a minimum. The source of the information on the spins of the final states for Figs. 1 and 2 was either the Nuclear Data Tables[3] or recent results.[4–7] The data on

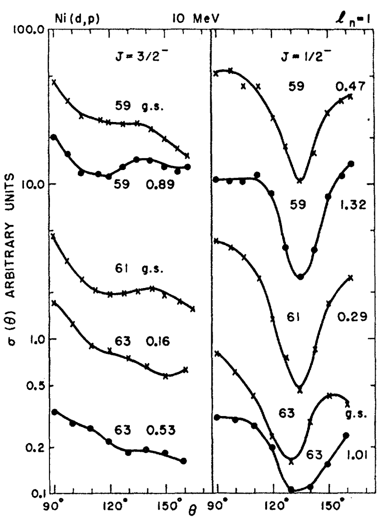

FIG. 2. Angular distributions in the backward quadrant of the (d,p) reaction for $l=1$ transitions to final states of known spin in the Ni isotopes.

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† On leave of absence at Princeton University, Princeton, New Jersey.

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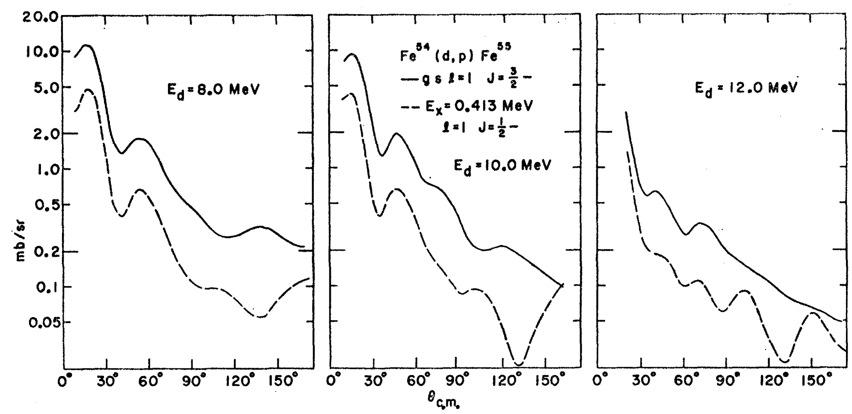

FIG. 3. Measured angular distributions of the $Fe^{54}(d,p)Fe^{55}$ reaction at 8, 10, and 12 MeV leading to the $J=\frac{3}{2}$ ground state and $J=\frac{1}{2}$ first excited state.

Cr are those of Green *et al.*,[8] who independently noted the backward difference between the two spin states.

To show that this difference is not sensitive to energy, Fig. 3 shows the angular distributions for the $Fe^{54}(d,p)$-Fe^{55} reactions at 8, 10, and 12 MeV; the effect can be seen to persist. Since all the nuclei considered here have $40<A<64$, one would like to know whether such a difference also persists for lighter nuclei. Figure 4 shows the angular distributions for two $l=1$ states in Mg^{25}. The one at 3.40 MeV is known to have $J=\frac{3}{2}$ and does not show a minimum at back angles. The 4.268-MeV state exhibits a sharp minimum which suggests $J=\frac{1}{2}$.

The only definite observed violation of this J dependence is in the $Ca^{42}(d,p)Ca^{43}$ reaction to the 0.39-MeV $J=\frac{3}{2}$ state of Ca^{43} which shows a minimum at $\sim 120°$. This, however, is a weakly excited state with a peak cross section only 0.05 times that of strong $l=1$ states. It is therefore possible that compound-nucleus effects could dominate the angular distribution at backward angles where the cross section is small.

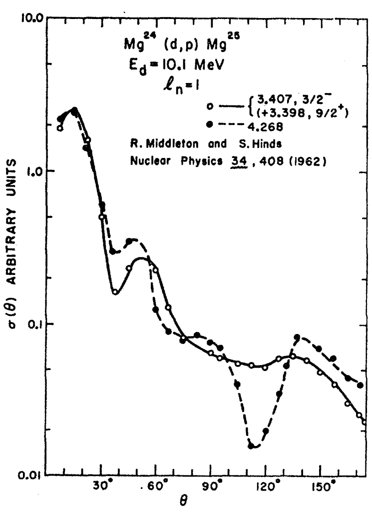

FIG. 4. Angular distributions of the $Mg^{24}(d,p)Mg^{25}$ reaction for the two strong $l=1$ transitions. The 3.403-MeV state is known to have spin $\frac{3}{2}^-$, the unresolved $\frac{9}{2}^-$ state is not believed to contribute appreciably. The upper state may be $\frac{1}{2}^-$ since no other strong $l=1$ transition is seen. The data are from Ref. 10.

Since the evidence for this J dependence is entirely from spin-zero target nuclei, it is of interest to examine other cases. This can be done indirectly by the $Fe^{56}(p,d)$-Fe^{55} reaction which would correspond to the $Fe^{55}(d,p)$-Fe^{56} reaction, with Fe^{55} prepared in its different states with various spins. The results are shown in Fig. 5 and the backward difference between $J=\frac{3}{2}$ and $J=\frac{1}{2}$ is still apparent.[9] The effect would clearly persist for nonzero target spin, where only one l is possible.

III. J DEPENDENCE IN $l=2$ TRANSITIONS

The evidence for $l=2$ transitions must come from nuclei in those regions of the periodic table where the d states of the shell model are near the Fermi surface. Such nuclei occur in the regions of atomic weight 16–40 and again at 75–100 and correspond to the filling of the $1d$ and $2d$ orbitals, respectively. A considerable amount of experimental information has been accumulated in the region of the $1d$ shell, but unfortunately most of it is restricted to angles smaller than 50°.

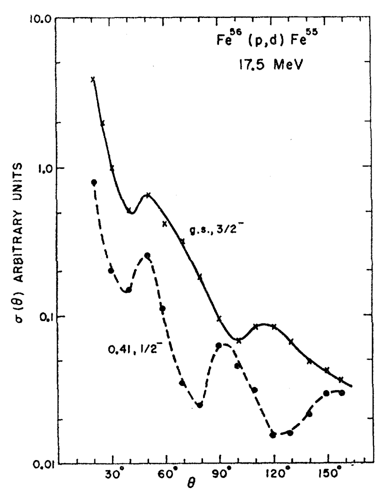

FIG. 5. Angular distributions of the $Fe^{56}(p,d)Fe^{55}$ reaction to the ground state and first excited state. The errors range from 5 to 30%. The data are those of Ref. 9.

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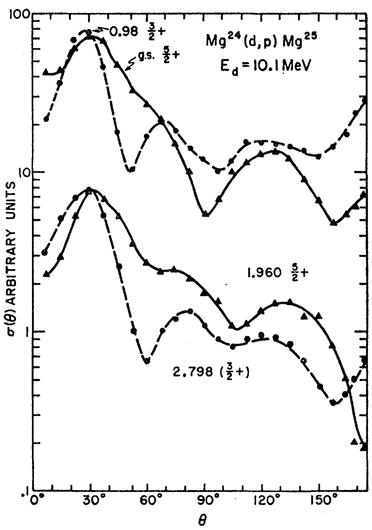

FIG. 6. Angular distributions of the $Mg^{24}(d,p)Mg^{25}$ reaction for $l=2$ transitions. The spins of the final states are indicated on the figure. The data are those of Ref. 10. The curves are displaced along the vertical scale to coincide in pairs at the forward peak.

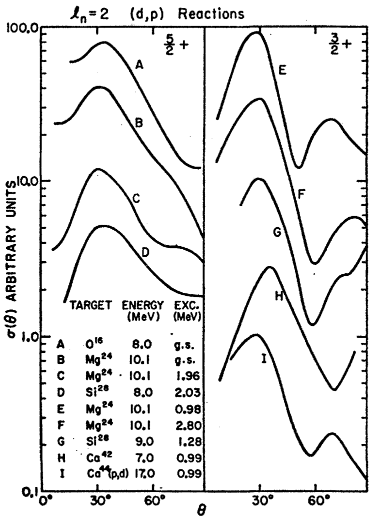

FIG. 7. Angular distributions of the (d,p) reaction for $l=2$ transitions leading to final states of known spin. The sources of the data are: A, Ref. 11; B, C, E, F, Ref. 10; D, G, Ref. 12; H, Ref. 13; and I, Ref. 14. The vertical scale is adjusted to avoid overlap between curves.

Complete angular distributions for the $Mg^{24}(d,p)Mg^{25}$ reaction were obtained by Middleton and Hinds.[10] Four of these show $l=2$ angular distributions and go to states of known angular momentum. These are shown in Fig. 6. No systematic effects appear in the backward direction or in the variation at extreme forward angles. The angular range between 40° and 80°, however, does behave differently for the two $\frac{3}{2}$ states than for the two $\frac{5}{2}$ states. The angular distributions for the $\frac{3}{2}$ states dip down sharply from the maximum at $\sim$30° to a minimum at $\sim$55° and come up to a secondary maximum at $\sim$70°, while those for the $\frac{5}{2}$ states fall off smoothly. Since Mg^{25} is believed to be a highly deformed nucleus, it is of interest to explore other more spherical nuclei as well. Information found[11–14] for nuclei ranging from O^{16} to Ca^{44} is summarized in Fig. 7; this behavior persists in all cases. This suggests strongly that the effect is a truly J-dependent one. It is of interest to note that compound-nucleus effects do not seem to alter this qualitative difference. Kuehner, Almqvist, and Bromley[15] have studied the $Si^{28}(d,p)Si^{29}$ angular distributions at a number of deuteron energies. Their results are shown in Fig. 8 in which it is clear that while compound effects cause the angular distribution at backward angles to fluctuate strongly with energy, the $\frac{3}{2}$ distribution always dips significantly lower at $\sim$55° than does the $\frac{5}{2}$ angular distribution. It is interesting to note that this difference seems to diminish at higher energies; the 15-MeV results of Blair and Quisenberry[16] are plotted along with the 8-MeV ones of Holt and Marsham,[12] in Fig. 9. It would appear that this effect is most pronounced for deuteron energies of 8–10 MeV.

Very little is known about the spins of states in the $2d$ shell. Differences that were tentatively attributed to a dependence on J have been observed in various $l=2$ transitions in $Se(d,p)$ reactions induced by 7.8-MeV deuterons.[17] However, the spins for these states were not known. There appear to be some differences in $l=2$ transitions observed in $Zr(d,p)$ reactions at 15 MeV.[18] These differences are not entirely consistent with the tentative spin assignments the authors made on the basis of spectroscopic factors. A possible difference between $\frac{5}{2}$ and $\frac{3}{2}$ states in this same reaction at 12-MeV

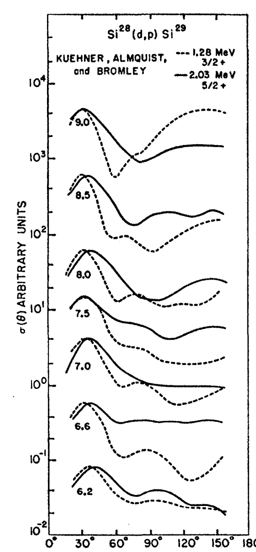

FIG. 8. Angular distributions of the $Si^{28}(d,p)Si^{29}$ reaction at various energies. These illustrate the persistence of the qualitative difference between the two types of angular distributions corresponding to different spins of the final state. The data are those of Ref. 15. The vertical scales are adjusted so that the corresponding peak cross sections coincide.

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[11] E. J. Burge, H. B. Burrows, W. M. Gibson, and J. Rotblat, Proc. Roy. Soc. (London) **A210**, 534 (1952).
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[16] A. G. Blair and K. S. Quisenberry, Phys. Rev. **122**, 869 (1961).
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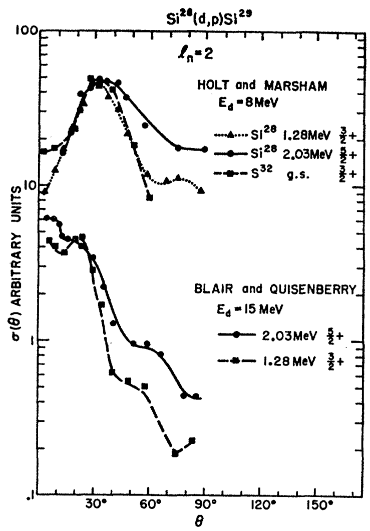

FIG. 9. Angular distributions for the $Si^{28}(d,p)Si^{29}$ reaction to the same two final states as in Fig. 8 at 8 MeV (Ref. 12) and 15 MeV (Ref. 16). The vertical scales are adjusted so that the corresponding peak cross sections coincide.

has been observed at backward angles; the data are shown in Fig. 10; the spin assignments are tentative ones suggested by Ref. 18. More experimental information is needed in the $2d$ shell; in particular, more spin measurements would be extremely desirable.

IV. J DEPENDENCE IN $l=3$ TRANSITIONS

Most of the evidence for such an effect must come from the region of nuclei around atomic weight 50 where the $1f$ shell is filling. A J dependence has been observed at forward angles in the (p,d) reaction with 28-MeV protons.[19] This would correspond to about 18-MeV deuterons. Backward angular distributions from the (d,p) reaction with 10-MeV deuterons are shown in Fig. 11. These results suggest a slight difference; the angular distributions for the $\frac{7}{2}$ states show a decreasing cross section between 100° and 160° while the $\frac{5}{2}$ ones are relatively flat. Unfortunately the spin-orbit splitting between the $f_{7/2}$ and $f_{5/2}$ states is about 6 MeV, and therefore there are no cases in which $\frac{5}{2}$ and $\frac{7}{2}$ states with measured spins and large (d,p) cross sections occur at low excitation energies in the same nucleus.

The (p,d) reaction has the advantage that in nuclei whose $f_{7/2}$ shell is closed and whose $f_{5/2}$ shell is beginning to fill, final states with spins of both $\frac{5}{2}$ and $\frac{7}{2}$ will be populated with relatively large cross sections and will occur at low excitation energy. The $Fe^{56}(p,d)Fe^{55}$ reaction was studied at 17 MeV, the angular distributions[9] being as shown in Fig. 12. It is clear that the two angular distributions corresponding to the two spin states are qualitatively different. The deuteron energy in this case is about 7.5 MeV. Similar results were obtained for a pair of states in the $Ni^{60}(p,d)Ni^{59}$ reaction.

For heavier nuclei, the only evidence is that of Erskine and Siemssen[20] who found in the $W^{182}(d,p)W^{183}$ reaction at 12 MeV that two states with $J=\frac{7}{2}$ showed a slight forward peak while two nearby $J=\frac{5}{2}$ states did not. Their data are shown in Fig. 13.

IV. DISCUSSION

The evidence presented here seems to indicate clearly that complete angular distributions of the (d,p) reaction may allow one to assign not only the transferred orbital angular momentum but also the total angular momentum. The empirical rules established at this time are as follows.

1. For $l_n=1$ transitions, deuteron energies between 7 and 12 MeV, $40<A<65$, and spectroscopic factors greater than about 0.1, the $J=\frac{1}{2}$ states exhibit a sharp minimum somewhere in the range $90°<\theta<145°$; but $J=\frac{3}{2}$ states do not.

2. For $l_n=2$ transitions, deuteron energies between 7 and 10 MeV, $16<A<44$, and spectroscopic factors greater than about 0.1, the $J=\frac{3}{2}$ states exhibit a sharp dropoff and a minimum at $\sim 55°$; the $J=\frac{5}{2}$ states do not.

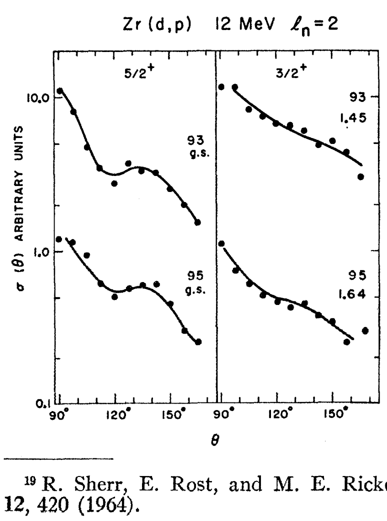

FIG. 10. Angular distributions of the (d,p) reaction on Zr isotopes for $l=2$ transitions to final states with probable spin assignments. The vertical scales are adjusted so that the corresponding peak cross sections coincide.

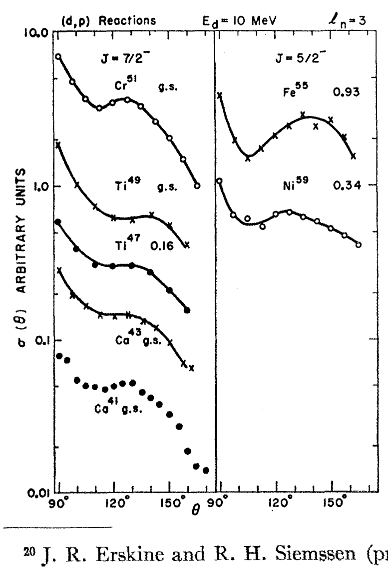

FIG. 11. Angular distributions of the (d,p) reaction with $l=3$ and final states of known spin. The vertical scales are adjusted to avoid overlap between curves for different targets.

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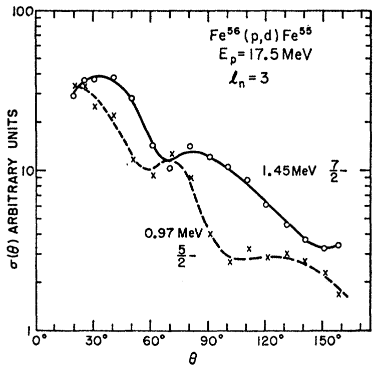

FIG. 12. Angular distributions of the $Fe^{56}(p,d)Fe^{55}$ reaction to two states with $l=3$. The errors range from 5 to 30%. The data are those of Ref. 9. The vertical scales are adjusted so that the corresponding peak cross sections coincide.

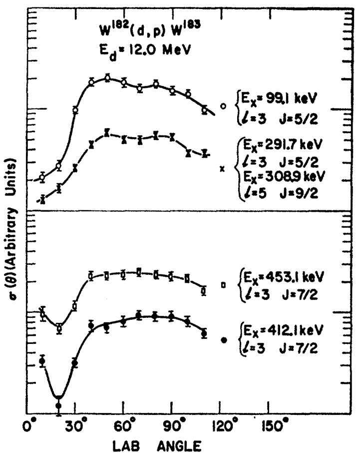

FIG. 13. Angular distributions for the reaction $W^{182}(d,p)$-W^{183} to states of known spin with $l=3$. Excitation energies and spins are as indicated. The data are those of Ref. 20.

3. For $l_n=3$ transitions, deuteron energies between 7.5 and 10 MeV, $40<A<60$ and spectroscopic factors greater than 0.1, the $J=\frac{5}{2}$ states decrease by less than a factor of 1.5 between 100° and 160° while the $J=\frac{7}{2}$ states decrease by more than a factor of 2.5. Additional J-dependent features at forward angles and higher energies were reported in Ref. 19.

The first rule has considerably more evidence to support it than the other two. It is thought that spin assignments based on it can be accepted with considerable confidence, while those based on the other two rules should be regarded as less certain for the present.

Cases of J dependences outside these regions are still too sporadic for the effect to be regarded as well established. Clearly more experimental investigations are needed. Measurements of spins by other techniques and measurements of complete (d,p) angular distributions at other energies and for different targets will help to put these rules on a more firm foundation. It may well be that at higher energies, where the compound nucleus cross sections become low, J-dependent effects will also be more reliable for the weaker states with spectroscopic factors less than 0.1.

In principle, the J-dependent effect can have its origin in three places: Spin-dependent forces can act on the incident deuteron, the captured neutron, and the outgoing proton. Sherr, Rost, and Rickey found that the forward-angle dependence in the 28-MeV (p,d) experiments for $l=3$ transitions could ge fitted with a distorted-wave Born approximation (DWBA) calculation and that it was principally caused by the spin-orbit term acting on the captured neutron.[19] The origin of the effects reported on here is not yet understood in detail in terms of the DWBA formalism.[21] A recent note by Greider[22] gives explanations for some of the observed J-dependent effects in terms of a diffraction model with a spin-orbit term.

Fulmer and Daehnick[23] have found that at backward angles the (d,t) reaction showed a J-dependent effect similar to that seen in the (d,p) reaction. This may mean that (d,He^3) and (He^3,d) reactions may also exhibit such effects. It also seems likely that results similar to the ones seen in (d,p) and (p,d) reactions will be seen in (d,n) experiments. In any case, if these J-dependent effects should turn out to be generally present and reliable, then the value of stripping and pickup reactions as the basic tool of nuclear spectroscopy will be greatly increased.

ACKNOWLEDGMENTS

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